Class ASTR 1050 Survey of Astronomy

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Date December 5, 2021

Title Instructions for Star Cluster Photometry

# Part I Objectives

In this assignment you will determine the brightness of stars in a cluster from images taken through various filters. This will allow you to estimate the temperature of the stars and from the hottest stars that remain, the age of the cluster.<sup>1</sup>

This assignment addresses the following minimum student outcomes in the syllabus, with the specific assignment items in the underlying lists:

### 1 Outcomes

- Relate fundamental concepts of physics, to include Newton's laws of motion, the Law of Universal Gravitation, the various radiation laws, spectroscopy, and the theories of special and general relativity, to problems in astronomy.
  - Inverse square law of reduced radiation intensity with increasing distance.
  - Diffraction pattern of star images taken through a telescope.
  - Photons converted to electrons in a CCD image, which are counted to determine radiation intensity.
- 2. Evaluate the methods used to measure the distances to celestial objects.

- Use known stellar magnitudes to determine magnitudes of unknown objects or variable stars.
- Recognize apparent magnitudes as an important component of determining distance if absolute magnitudes can be established.
- Recognize that apparent magnitudes infer absolute magnitudes if measured stars are all at the same approximate distance from Earth.
- 8. Defend theories regarding the evolution of stars of various masses.
  - Compare intrinsic luminosity of stars of similar distance
  - Since stars in a cluster are at the same distance from Earth, their apparent magnitudes are directly related to their luminosity.
  - Reveal evolutionary paths of stars of various masses on an H-R diagram

Figure 1: H-R diagram with evolution paths for stars of various masses



<sup>&</sup>lt;sup>1</sup>Graphics not generated by the author and not cited explicitly were used with permission from the course textbook.[2]

- 9. Interpret a Hertzsprung-Russel diagram.
  - Measure magnitudes of stars in certain color wavelength bands
  - Relate color magnitudes to the peak wavelength of a black body and thus obtain temperature
  - Relate apparent magnitudes to absolute magnitudes or luminosity for objects at the same distance from Earth
  - Plot numerous stars in a cluster on a H-R diagram to reveal their current stage of stellar evolution

Figure 2: H-R diagram with regions for stars in different stages of evolution



## 2 Theory

Our objectives and process rely heavily on the theory of stellar evolution and the observation

of many stars in various stages of evolution. It also limits some variables that could confound our measurements and make analysis difficult. Here are some key points:

### 2.1 Why clusters?

Since star clusters form from the same molecular cloud, they are roughly the same distance from us. Their spread is insignificant to the enormous distance from Earth to the center of the cluster. This eliminates the confounding factor of light intensity falling off by the square of the distance. We can say that the apparent difference in light intensity of stars in the cluster represents true differences in luminosity.





### 2.2 Color and temperature

The Planck black body radiation curve grows in intensity for all wavelengths as temperature increases, but also shows a wavelength shift in peak intensity with temperature. This explains why hot objects glow first deep red, then orange, yellow, and blue as they get hotter and brighter. Wein's Law allows us to calculate the temperature of a black body based upon the peak color of the black body curve measured at all wavelengths. Planck developed this curve empirically, and it was only with the discovery of quantum mechanics that the physical processes behind this relationship became clear.





Figure 5: black body curve for hot stars in a young cluster



Note that the overall curve is higher and wider for the hotter star. The curve *does not* shift toward the blue end, making the red end dimmer, but disproportionately increases the blue wavelengths, skewing the curve.

### 2.2.1 Asymmetric curve

Since the black body curve is asymmetric, it has a distinctive shape for each curve produced by objects at different temperatures. We can deduce the shape of the overall curve by establishing the shape of just a segment of the curve. The slope for the curve through two points changes with not only the colors, but the intensity of the black body radiation. This combination of effects allows us to determine not only the color, but the temperature of a black body source.

### 2.2.2 Color filters

Measuring the light intensity of stars through two or more color filters provides more or less crude estimates of the shape of its light emission curve. Why use them when spectroscopy provides so much more information? Filters are easy to apply and change when gathering images through a telescope with a monochrome or black-and-white camera. They also take far less time to gather data and can use smaller telescopes. Since large telescope time is in short supply, astronomers rely on this simple approach to collecting useful data, and apply more processing and analysis to the results to extract meaningful information.

The filters used to collect the images we are using in this laboratory assignment were developed by astronomers many years ago and there now exists a huge body of data on stars taken with these same filter pass bands. The U filter examines the ultraviolet range, while the I filter excludes all but the infrared range. In our data set we have an image in the H filter range, which is in the far infrared. The figure below shows the wavelength in Angstroms for each of the five most popular filters. [6] Note that one Angstrom equals to 10 nanometers.

#### Figure 6: Johnson/Cousins BVRI Filters



#### 2.2.3 Stars as black bodies

Stars are not perfect black bodies, since they have emission and absorption lines in their spectra, and on a detailed level their light intensity curves are not smooth like those of purely thermally produced light sources. However, reducing the level of detail does reveal a close approximation to a black body curve, and we can estimate the star's temperature by the shape of its light intensity curve, within certain temperature limits.

Due to the inverse square law of radiation fall-off with distance, astronomers search for ways to eliminate the need for knowing the distance to stars. As described above, choosing a cluster and assuming the same distance of member stars is a convenient way to study the evolution of a group of stars. However, dividing the intensity readings in one filter bandwidth with intensities from another filter bandwidth eliminates the distance fall-off factor, even for stars not in the same cluster or local area. This allows us to compare the colors of stars at all distances.

This does, however introduce a complication when you want to deduce temperature from the black body curve. The resulting ratio of intensities is a transcendental equation. [11] We cannot use this ratio of intensities to find temperature, which is our goal. We must resort to numerical approximations or fit measurements to Planck's black body curve. However, we can derive an analytical formula for temperature via a mathematical study of Planck's law. F.L. Ballesteros did that and derived an equation, which will be shown in detail below. For a detailed account of how Ballesteros derived this formula, see his journal article. [1]

# 2.2.4 Temperature and speed of stellar evolution

Hot stars use up their nuclear fuel in their cores faster than cooler stars, and thus evolve into giant stages sooner than cooler stars. Typically the hottest stars maintain hydrogen core fusion only for a few million years, and then develop hydrogen shell and helium fusion in their cores, thus evolving off the main sequence of the H-R diagram to become blue and red giants and super giants.



### 2.2.5 Maximum temperature and age

If we can determine the color of stars in a cluster, we can deduce how old it is by noticing which star temperatures are missing. As a star cluster ages, the hotter stars evolve into cooler giant stars and the maximum temperature of stars in the cluster decreases.

Relatively young clusters still retain their hot, massive stars, and thus these still occupy the main sequence in the H-R diagram. Below is the H-R diagram of the relatively young Hyades open cluster in Taurus, showing relatively hot stars at the Turnoff point.

Contrast this plot with the H-R diagram of the old globular cluster Omega Centauri, showing few hot stars and more red giant stars, where all the hot stars have evolved into red giants and white dwarfs. You can see that the turnoff point has moved to cooler temperatures, perhaps 7000 K. Figure 8: H-R diagram of young cluster



Note that the stars in the 10,000 K area of the main sequence for the old cluster are possibly what astronomers call "blue stragglers" because they have received additional material from companion stars in close binary systems, and thus have begun to burn hotter as their mass increases. This happened billions of years *after* the cluster formed, however.

Figure 9: H-R diagram of old cluster



### 2.3 Plots and patterns

Plotting measurements for individual stars in a cluster can reveal patterns which characterize the evolutionary state and temperature of stars, and thus the age of the cluster. Using star color indexes as coordinates will allow us to construct these diagrams easily.

#### 2.3.1 Color index

The relative difference between measurements of a particular star at different color wavelength bands provides a convenient way to characterize the shape of the intensity curve. We can subtract the magnitude taken through a green filter (V) from one taken through a blue filter (B), for instance. In our filter system, this is often shown as the B-V magnitude difference. We can calculate the difference between other color filters as well, such as between green (V) and red (R) which is V-R, or even between green (V) and infrared (H), which would be V-H.

Since we are subtracting one magnitude from another, that is the same as dividing the photon counts, due to magnitudes being a logarithmic quantity. What this means is that the effects of distance are divided out, and we have a way to compare stars of varying distances from Earth directly without calculating their luminosity or absolute magnitude. That means we don't need to know how far away they are, making the analysis a lot easier.

Since a color index value captures the slope of the line segment on the black body curve for two points, one at each color measurement wavelength, we can define the shape of the black body curve associated with the color index value. Black body curves produced at different temperatures are unique and correspond with color index values in a one-to-one relationship.

This allows us to estimate the peak intensity of the particular black body curve associated with that color index, and by Wein's Law calculate the temperature of the star. This is a surprisingly good estimate of temperature of main sequence stars within a restricted range of temperatures similar to the Sun.

Figure 10: black body curves of various temperatures and slope of line connecting B and V magnitude points



### 2.3.2 Color index and temperature

Color index values can be converted to temperature through a mathematical formula.

For a perfect black body, this relationship follows the formula  $CI = B - V = -0.71 + \frac{7000}{T}$ .

Note that for a 10000 K star, the CI = 0, which happens to be how the system is calibrated and corresponds to a hot star like Vega, also known as Alpha Lyra.

Since stars are not perfect black bodies, for cooler Sun-like stars in the 4000 to 10000 K range, a better approximation is  $CI = B - V = -0.865 + \frac{8540}{T}$ 

If we want to calculate temperature from the CI, we just perform some algebraic manipulation on the above formula:  $T = \frac{8540}{(B-V)+0.865}$ 

Ballesteros' formula [4] provides perhaps an even better approximation of temperature:  $T = 4600K \left(\frac{1}{0.92(B-V)+1.7} + \frac{1}{0.92(B-V)+0.62}\right)$ In the plot below, note that this is not a

In the plot below, note that this is not a linear relationship. At B-V=0 and less, the relationship curves upwards asymptotically, and our formula may not provide accurate results. Figure 11: Relationship between color index and star temperature



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### 2.3.3 Color-Temperature

By calculating temperatures with the above formula, we can plot a color index on the Y axis and temperature of the stars on the X axis. This is similar to a color-color plot, and substitutes the more familiar temperature scale on the horizontal axis. Note that hotter temperatures are on the left and the scale is logarithmic, a convention in astronomy that underlies the H-R diagram.





#### 2.3.4 Relative versus absolute

Using color indexes removes the need to standardize observed magnitudes into absolute magnitudes or luminosity. This removes some confounding factors. The need to compensate for the inverse square distance fall-off of intensity mentioned above is just one advantage.

However, astronomers must still calibrate their measurements to remove instrumental and atmospheric effects. This allows them to reference their measurements to a standard, and magnifies the usefulness of their efforts, as other astronomers may compare their measurements.

We will not calibrate our measurements to a standard system. Not only is this procedure beyond the scope of our lab exercise, we do not have all the necessary information about the telescope and observing conditions to perform such a transformation. We will still be able to extract meaningful information from our science images, though.

### 2.3.5 Interstellar Reddening

Starlight that travels through interstellar space encounters varying amounts of dust and gas in the interstellar medium. These particles selectively disperse light of short wavelengths, such as in the blue end of the spectrum. The light that reaches the observer on Earth is thus deficient in that part of the spectrum, and astronomers call this effect "reddening." This affects color indexes, and must be addressed if we want meaningful information such as temperature and the age of a star cluster.

Interstellar reddening, also called "extinction," changes not only with each cluster, depending on the presence of interstellar clouds in our line of sight to the cluster, but also changes with the color index. Since blue wavelengths are most affected, color indexes with blue filter components are affected more.

For our exercise involving NGC 957, we can use extinction factors derived from careful examination of the spectra of stars in the cluster. Figure 13: Starlight reducing the blue end of a black body curve and decreasing the strength of spectral lines



From such an analysis astronomers derived extinction factors for the B-V, V-R, and V-H color indexes, among others.[13] They presented ratios of extinction factors for color indexes to illustrate an "extinction law" and compared it to a "normal" configuration. They concluded that the differences to a normal extinction law were minor, and thus no special conditions affecting extinction exist along the line of sight to NGC 957.

Figure 14: NGC 957 color index ratios

Objects	$\frac{E(U-B)}{E(V-J)}$	$\frac{E(B-V)}{E(V-J)}$	$\frac{E(V-R)}{E(V-J)}$	$\frac{E(V-I)}{E(V-J)}$	$\frac{E(V-H)}{E(V-J)}$
Normal value NGC 637 NGC 957	$\begin{array}{c} 0.32 \\ 0.32 \pm 0.03 \\ 0.34 \pm 0.02 \end{array}$	$\begin{array}{c} 0.43 \\ 0.45 \pm 0.04 \\ 0.47 \pm 0.01 \end{array}$	$\begin{array}{c} 0.35 \\ 0.36 \pm 0.03 \\ 0.39 \pm 0.03 \end{array}$	$\begin{array}{c} 0.70 \\ 0.69 \pm 0.05 \\ 0.75 \pm 0.05 \end{array}$	$1.13 \\ 1.11 \pm 0.08 \\ 1.12 \pm 0.03$

This table does not list the extinction factors we need to correct for interstellar reddening, but we can derive them from the ratios provided in the table. The authors did provide the extinction factor for the B-V color index,  $E(B-V) = 0.71 \pm 0.05$ , which I used to reduce those ratios. I first derived the E(V-J) from the  $\frac{E(B-V)}{E(V-J)}$  ratio by dividing it into E(B-V), yielding

$$E(V-J) = E(B-V) / \left(\frac{E(B-V)}{E(V-J)}\right) = 0.71/0.47 = 1.511$$

I then multiplied E(V-J) times the ratios for  $\frac{E(V-R)}{E(V-J)}$  and  $\frac{E(V-H)}{E(V-J)}$ .

$$E(V - R) = E(V - J) \times \frac{E(V - R)}{E(V - J)}$$
$$E(V - R) = 1.511 \times 0.39 = 0.589$$

and

$$E(V - H) = E(V - J) \times \frac{E(V - H)}{E(V - J)}$$
$$E(V - H) = 1.511 \times 1.12 = 1.692$$

If we are to construct a color-magnitude plot similar to the H-R diagram we need the V magnitude for the vertical axis. This magnitude will suffer the effects of interstellar reddening and must be corrected to eliminate this dimming of the star by the interstellar medium. The amount we need to subtract from the observed magnitude, to make it brighter, is given by the following formula: [13]

$$R_V = 1.1 \frac{E(V - K)}{E(B - V)}$$

From the cited study of NGC 957, this value is

$$R_V = 1.1 \frac{2.00}{0.71} = 3.1$$

These calculations provide the extinction factors we needed. The table below summarizes these values:

Filter	Extinction
E(B-V)	0.71
E(V-R)	0.589
E(V-H)	1.692
$R_V$	3.1

We can use those factors to correct our readings from our filtered images, as outlined in the Process and Analysis sections below. Basically, we will subtract these factors from the color indexes, since a bluer color implies a lower number magnitude in the blue end, which results in a smaller color index value, or even a negative color index.

Reddening essentially adds to the color index values, shifting them up and to the right in a color - color plot. The figure below shows this via an arrow labeled  $A_V = 3$ , with  $A_V$  being a common abbreviation for the extinction in the V filter band. By subtracting these extinction values from the appropriate color indexes, we will remove that shift, moving the points to the left and down on the graph.

Figure 15: Effect of extinction [10]



### 2.3.6 Color-color plots

If we measure a star's magnitude in at least three color wavelength bands, we can plot it in a graph that uses one color index for the horizontal axis and the other color index for the vertical axis. This relieves us of the task of determining absolute magnitude of the stars, which is what the H-R diagram demands for its Luminosity vertical axis.

With practice we can interpret the patterns of points on the color - color plots to be groups of stars in various stages of evolution. Black bodies form straight lines, but stars are not perfect black bodies, so their patterns resemble curves with a dip in the middle. The following graph shows this for the Main sequence and Supergiant classes of stars.

Note that our color - color plots may look somewhat different, since we will be comparing V-R and V-H to B-V, rather than the U-V index shown below. Also, the U-B axis in this plot is inverted, which puts the blue end of the axis towards the top, and effectively flips the curves vertically.

Figure 16: Color-color plot with curves for stars in various stages of evolution [8]



Other photometric systems will yield color - color plots with slightly different shapes, so you have to get used to the system in which your data resides. Here is one using the Sloan Digital Sky Survey u'g'r'i'z' filter system.

Figure 17: Color-color plot in SDSS filter system with stars of various spectral classes.[9] Corresponds roughly to  $g' - r' \Rightarrow B - R$  and  $r' - i' \Rightarrow R - I$ 



## Part II Resources

This exercise will be performed with your computer and may require online access. Here is a detailed list of items you will need to download and install on your computer:

- AstroImageJ This is a special extension of the ImageJ program used in science to process and analyze science images in the .fits format. It contains useful tools for labeling target and comparison stars, defining reference magnitudes, analyzing star intensity profiles and establishing apertures for measuring star intensities and background contributions. It also extracts magnitudes and allows you to compare results using different combinations of comparison and check stars. You can then save measurements for later analysis in a spreadsheet application.
- SalsaJ This application performs some basic photometry functions, and may be used as an alternative to AstroImageJ if necessary. It does require more labor in a spreadsheet, as it reports only star intensities, not magnitudes. It should only be used as a last resort if AstroImageJ will not install on your computer.
- NGC 957 filtered images This is an open star cluster which formed from the same molecular cloud of interstellar gas, and the members of this cluster are therefore all at approximately the same distance from Earth. Since they formed at about the same time, they are also all about the same age.

We have four images, one of each taken through different photometric filters, representing different wavelength bands. They are labeled B, V, R, and H, and represent the general colors blue, visual or green, red, and near infrared.

Each image contains information about the target and the image in a FITS header.

You can view this header and even edit it with new information. Here is a screenshot of the FITS header for the NGC 957 image.

Figure 18: NGC 957 FITS header



The star cluster is located in the constellation Perseus at a distance of 1815 parsecs, or 5920 light years. Its celestial coordinates are: RA: 02h 33m 21.0s DEC:  $+57^{\circ}$  33' 36". An Internet search for the age of NGC 957 produced the following information:[12]

Figure 19: Open cluster NGC 957

<b>Basic Parameters</b>				
Right Ascension (2000)	02 33 21			
Declination (2000)	+57 33 36			
Galactic longitude	136.287			
Galactic latitude	-2.645			
Distance [pc]	1815			
Reddening [mag]	0.842			
Distance modulus [mag]	13.90			
Log Age	7.042			
Metallicity				
Notes				

**PlotLy** PlotLy is an online plotting tool that simplifies the creation of color-color and color-magnitude diagrams. It provides a standard interface independent of your computer or operating system. Here is a link to the website: PlotLy Chart Studio

This tool allows you to import data from your spreadsheet and create the plot, then save it and the data grid for future use. You can export the plot to an image file that you can then use in your report. You can also share your plot via a URL address, such as this link for my plot: NGC 957 Color-Color Diagram

Here is the color-color plot of NGC 957 created in PlotLy Chart Studio:

Figure 20: NGC 957 B-V vs V-R Color-Color Diagram



# Part III Process

Determining the apparent brightness of stars in the filtered images requires us to perform aperture photometry, which was explained in detail in another document. Here we will focus only on the additional targets and images recorded in different filter wavelength bands. The technique is essentially the same as aperture photometry, and is just repeated for each of the filtered images.

To maintain consistent label names from one filtered image to the next, we must maintain the same order in placing our apertures in the image. AstroImageJ shows this order via a number appended to either a "T" designation for a target star or a "C" designation for a comparison star. The following figure shows one sequence that was chosen for the four filtered images for the open cluster NGC 957.





## 3 Calibration

The magnitudes we measure from our images are instrumental magnitudes, and contain the influences of the telescope, camera, and atmosphere. As such, these measurements are not directly comparable to other measurements made by other instruments at other locations and times. This limits the usefulness of these measurements.

To eliminate these limitations, astronomers transform the instrumental magnitudes into a standard system. They do this by removing the effects of the atmosphere and adjusting the magnitudes for the spectral response of the camera. They compare their instrumental magnitude measurements of standard stars with known calibrated magnitudes of those stars, and derive mathematical relationships that account for the differences. The key parameters that perform these transformations are called calibration coefficients.

We do not have measurements of standard stars in our data set, and thus cannot transform these measurements to a standard system. It may be that the astronomers that collected the data we are using at some point did collect instrumental magnitudes of standard stars and did in fact calculate the calibration coefficients, but they are not available to us. As such, we will have to limit our analysis to general conclusions and realize that any specific numbers we quote are subject to distortion by the instrument and technique.

## 4 Procedure

Please refer to the video tutorial for detailed instructions on how to perform this activity. Also refer to the script, which contains a log of the actions performed in the video. The script may be found in this file:

StarClusterPhotometryTutorialScript.pdf.

In short, our procedure follows these steps:

- 1. Prepare work environment
- 2. Perform multi-aperture photometry on filtered images in AstroImageJ
  - V filter green
  - B filter blue
  - R filter red
  - H filter infrared
- 3. Combine measured star magnitudes in all filters into a composite spreadsheet
  - Compute color index values by subtracting instrumental magnitudes
    - B-V
    - V-R
    - V-H
  - Apply a correction factor to the color indexes due to interstellar reddening.
  - Compute temperature from B-V color index for Sun-like stars
- 4. Create graphs in the online graphing tool PlotLy

- (a) Transpose data grid to meet PlotLy's format requirements
- (b) Create color color graphs using B-V and V-R color indexes
- (c) Create magnitude color graph using V magnitude and the B-V color index
- (d) Create magnitude temperature graph using V magnitude and temperature
- (e) Save graphs and data grid for later use and to share with others.
- (f) Export graphs for annotation and use in a laboratory report.
- 5. Annotate graphs in an image editor to reflect our analysis and application of theory to our results.

# Part IV Analysis

Now comes the interesting part of the project, where you actually glean vital information about the cluster from your measurements. You will be amazed at what we can infer from just these filtered images and apparent magnitude readings. This is where the true power of the color index vs color index and magnitude vs temperature relationships begins to shine.

Color indexes have the significant advantage in that they are independent of dimming by distance. We assume that both colors in the index dim by the same amount, and thus distance does not change their difference. We can compare all sorts of stars with color indexes and not worry about how far away they are from us, or how inherently bright the stars are. All we are interested in is their color differences.

### 5 Color vs color graph

A perfect black body changes color indexes proportionally as temperature increases, and will plot as a straight line on a color-color plot. Here is a plot of stars in the NGC 957 star cluster with a fitted linear regression line showing that this relationship holds pretty well, and thus verifying that stars are close to being perfect black body radiators.





Note that some points deviate significantly from the line. That might mean that they are obscured by dust or have strong emission lines in their spectra, reducing or enhancing specific wavelength bands. To understand how these phenomena affect the graph, we first have to clarify what the color index signifies.

### 5.1 Color index number

Color index numbers can be positive or negative. For a B-V color index, a negative value indicates a smaller number for the B filter magnitude than for V. Remember that magnitudes are smaller numbers the brighter the star, so a negative B-V color index indicates the star is stronger in the blue wavelengths than the V or green wavelengths. Conversely, a positive B-V color index means the star is not as blue, and more red in color. The same could be said for the V-R color index, since a larger V magnitude than R magnitude means it is dimmer in the green part of the spectrum than in the red.

Refer back to the color-color plot that shows the main sequence line for color gradients applied to the color index axes to illustrate this relationship. I used the *Krita* graphics editor to apply the annotation and color gradients.

Interstellar dust scatters blue light more than red, so it makes stars appear more red to observers. This shifts their plotted positions on the color-color plot up and to the right. Astronomers call this effect "interstellar reddening." We can explain some of the points that deviate from the straight line perfect black body line by reddening effects, as that would pull the points either up or to the right, or a mixture of both directions.

### 6 Color Magnitude plot

If we substitute magnitudes for one of the color indexes on the vertical axis, we now have a way to compare the apparent brightness of stars at various colors. This normally wouldn't tell us much, except for the fact that in a star cluster, all the stars are at approximately the same distance from the Earth, and thus we can infer their absolute magnitudes, or luminosity, from the apparent magnitudes.

Here is a color magnitude plot that shows the brightest stars toward the top, and the reddest stars toward the right, just as in the H-R Diagram.

Figure 23: Color - Magnitude Plot



### 6.1 Temperature - Magnitude Plot

Since we can directly relate the B-V color index to temperature in the star's atmosphere, we can transform our color - magnitude plot into a temperature - magnitude plot. This conveys information that we can use later on to determine the star's spectral type and its mass. The shape of the plot doesn't change significantly.

Just the horizontal axis and units change. Note that the hotter temperatures are on the left, which corresponds with the blue end of the color spectrum, and the smaller and even negative B-V color index numbers. Also, the temperature scale is logarithmic, which matches the convention in the H-R diagram, and places equal emphasis on cool and hot temperature ranges.





In a similar way to the H-R diagram, we can interpret different regions with different stages of stellar evolution. The following plot shows these areas marked for stars in the NGC 957 star cluster.





textitNote that these designations are speculative and illustrate possible interpretations. They should not be considered actual conditions of member stars, but illustrative of the concepts only. More definitive interpretation would require calibrated photometry data that removes instrumental and observational distortions.

### 7 Age of cluster

Observing the maximum temperature in the above Magnitude vs Temperature plot provides information we can use to estimate the age of the star cluster. Realizing that massive stars burn hotter and exhaust the hydrogen in their cores sooner, we can deduce that some of those stars will have already evolved off the main sequence. They may be the stars occupying the area labeled "Evolving Branch" in the above diagram, or even the "Red giant" star.

The remaining stars on the main sequence are cooler and longer-lived. Since all stars in the cluster formed at about the same time, we can say that these remaining stars provide clues to the age of the cluster. If we can determine the point in the diagram where stars are just starting to evolve off the main sequence, that will indicate the age of the cluster. Astronomers call this point the *Main Sequence Turn-Off*, or MSTO point.

Once we have the MSTO point, we can use the relationship between mass, temperature, and expected lifetime on the main sequence to estimate the time since formation for the cluster to the turn-off point. The following table shows the estimated lifetime of stars on the main sequence based upon their spectral class and temperature. We can compare the maximum temperature in our main sequence stars for NGC 957 to the listed temperature ranges for stars in the table and read across the appropriate row in the table to read the time on the main sequence.

The following figure shows the color index values, V instrumental magnitudes, and tem-

peratures derived from the B-V color index. The star labels correspond to the aperture names in the original filtered image in AstroImageJ.

Figure 26: Color Indexes, Magnitudes, and Temperatures

Label	B-V	V-R	V-H	V Magnitudes	Temp K-(Ballesteros)
т1	-0.1932	-0.5248	-0.5876	8.8	13423
C2	-0.1200	-0.5791	-0.5119	8.8	11920
C3	-0.1380	-0.3783	0.2001	8.1	12254
T4	0.0275	-0.4038	-0.5579	11.6	9795
T5	-0.0165	-0.3965	-0.5234	11.6	10336
T6	-0.0369	-0.4341	-0.4904	10.0	10611
T7	0.0030	-0.4088	-0.5406	11.8	10087
T8	0.8388	0.1538	0.2109	9.7	5166
Т9	-0.0200	-0.4184	-0.6257	11.8	10382
T10	0.3575	-0.1846	-0.1400	9.8	7115
T11	0.3170	-0.2079	-0.2654	11.4	7355
T12	0.5268	0.0257	0.1581	11.7	6270
T13	-0.0769	-0.5004	-0.5721	9.1	11199
T14	0.2364	-0.2748	-0.4025	11.7	7892
T15	-0.1004	-0.4438	-0.5488	11.1	11580
T16	-0.0979	-0.5030	-0.5966	10.5	11538
T17	-0.0037	-0.4537	-0.5867	11.9	10172
T18	0.2956	-0.2446	-0.2265	10.5	7490
T19	-0.0377	-0.4592	-0.5598	11.1	10621
T20	-0.2136	-0.5548	-0.6182	8.8	13921
T21	0.1869	-0.2867	-0.3518	10.6	8266
T22	-0.0613	-0.4644	-0.5591	10.7	10960

Here is a table that shows time on the main sequence for stars of various spectral classes and surface temperatures.[3] This table combines information from various sources to produce the main sequence lifetime of stars of various masses using the formula

$$t_{MS} = t_{Sun} \times \frac{M}{L(M)}$$

where  $L(M) = M^{3.5}$  is the luminosity function when M is the mass in solar masses. This resolves to the following formula:[5]

$$t_{MS} = 10^{10} years \times \frac{M}{M^{3.5}}$$
$$= 10^{10} years \times M^{-2.5}$$

Here is another table that provides a finer breakdown in temperatures, but omits the spectral classes.[7]

Lifetimes of Main Sequence Stars					
Spectral	Surface	Solar	Time on		
Type	Temp K	Masses	M.S. (yr)		
		(Sun=1)			
O5	54,000	40	$1 \times 10^{6}$		
B0	29,200	16	$10 \times 10^{6}$		
A0	9600	3.3	$500 \times 10^{6}$		
F0	7350	1.7	$2.7 \times 10^9$		
G0	6050	1.1	$9 \times 10^{9}$		
K0	5240	0.8	$14 \times 10^9$		
M0	3750	0.4	$200 \times 10^9$		
K0 M0	5240 3750	0.8 0.4	$     \begin{array}{r}       14 \times 10^9 \\       200 \times 10^9     \end{array} $		

Table 1: Main Sequence Stellar Lifetimes

Figure 27: Temperature and Time on M.S.

Key Properties of Main Sequence Stars						
Mass/M <sub>Sun</sub>	Luminosity/L <sub>Sun</sub>	Effective Temperature (K)	Radius/R <sub>Sun</sub>	Main sequence lifespan (yrs)		
0.10	3×10 <sup>-3</sup>	2,900	0.16	2×10 <sup>12</sup>		
0.50	0.03	3,800	0.6	2×10 <sup>11</sup>		
0.75	0.3	5,000	0.8	3×10 <sup>10</sup>		
1.0	1	6,000	1.0	1×10 <sup>10</sup>		
1.5	5	7,000	1.4	2×10 <sup>9</sup>		
3	60	11,000	2.5	2×10 <sup>8</sup>		
5	600	17,000	3.8	7×10 <sup>7</sup>		
10	10,000	22,000	5.6	2×10 <sup>7</sup>		
15	17,000	28,000	6.8	1×10 <sup>7</sup>		
25	80,000	35,000	8.7	7×10 <sup>6</sup>		
60	790,000	44,500	15	3.4×10 <sup>6</sup>		

Suppose we determine the maximum temperature to be the star on the extreme left in the "Main Sequence" area on the graph above. We can scan down the table of data values to find the calculated temperature, which in this case is 13921 K. Now we can estimate the age of the cluster by finding the appropriate main sequence time in the table below. We can see that our temperature falls between the B0 and A0 spectral classes, with corresponding ages of  $10 \times 10^6$  and  $500 \times 10^6$  years, or 10 and 500 million years.

If we want a more precise value, we can interpolate between these values based upon the proportional difference between the two bounding temperatures and our measured temperature. This also happens to use the point-slope equation form for a straight line you learned in basic algebra, or  $y = y_1 + (x - x_1) \times \frac{y_2 - y_1}{x_2 - x_1}$ . Since both the temperature and time on M.S. column values appear to be exponentially spread out, we should use logarithms. Here is the calculation using values from the second more precise table:

$$log(t_{MS}) = log(2 \times 10^9) + (log(13921) - log(7000)) \times \frac{log(2 \times 10^8) - log(2 \times 10^9)}{log(11000) - log(7000)}$$

= 7.7801 log years or  $10^{7.7801} = 6.0 \times 10^7$  years, which is 60 million years.

Referring back to the table of information on NGC 957 shown in the Resources section of this document, the published log age of NGC 957 is 7.042, which can be converted to years by using the logarithm as the exponent of 10:

$$t_{MS} = 10^{7.042} = 11.02 \times 10^6 years$$

or only 11 million years.

Our estimate is off by a factor of about 6 times. What could cause such an error? Here is a list of possible factors:

- **Calibration** Recall that our magnitudes are not calibrated to a standard system, and thus may be subject to loss of sensitivity in the shorter and bluer wavelengths due to the camera spectral response.
- **Temperature calculation** Our mathematical relationship for calculating temperature from the B-V color index may be only accurate for Sun-like stars with surface temperatures no greater than 10,000 K. Hot stars with main sequence lifetimes of only 10 million years are class B0 stars with surface temperatures of almost 30,000 K. Our calculation of temperature might not be accurate for stars that hot.

Indeed, if you refer back to the relationship between color index and star temperature figure near the beginning of this document, you can see that the curve is very nonlinear for stars with a B-V color index close to or less than zero. Indeed, the B-V color index for this star, which is T20 in our table of star measurements, is -0.2136, which is less than zero and in the nonlinear range.

# Part V Conclusion

In this document we present the theory of using photometry to determine critical parameters of star clusters. We explore an example of multiple aperture photometry applied to several intermediate wavelength band filtered images of the star cluster NGC 957.

We list the resources and general procedure astronomers use to calculate instrumental magnitudes from images of stars and use them to form color indexes, which are independent of light dimming due to distance. We create color - color diagrams that exhibit certain traits of black body radiation and stellar evolution.

We also create graphs showing color index versus magnitude. This is similar to the H-R diagram, with respect to the member stars in the cluster. Although the H-R diagram uses absolute magnitude for its vertical axis, and our magnitudes are apparent magnitudes only, we can infer the luminosity of these stars since they are all at approximately the same distance from the Earth.

We also use a relationship between stellar atmosphere temperature, as determined by the B-V color index, and mass to derive the time on the main sequence for the hottest star measured. This gives us an estimate of the age of the star cluster.

Due to limitations in our data and our introductory scope in this assignment, we do not calibrate the instrumental magnitudes into a standard system, and thus our calculated temperatures and cluster age are inaccurate. However, we are able to explore the concepts of stellar evolution, color and temperature, and mass which determine the time on the main sequence for member stars.

This exercise should acquaint us with the methods and principles astronomers use to extract information about stars using photometry. It should also inspire appreciation and respect for the degree of effort and rigor necessary to make meaningful statements about objects so distant and mysterious to mere mortals on Earth.

# Part VI Appendix

## 8 SalsaJ

### 8.1 Magnitude calculation

SalsaJ reports the star intensity, not the magnitude. It also does not have a way to input reference star magnitudes. This requires the user to calculate magnitudes from the definition that involves the  $log_{10}$  formula. Here is the formula for calculating the magnitude of a target star given the magnitude of a reference star:  $mag_{target} = mag_{reference} - 2.512 \times log \left(\frac{flux_{target}}{flux_{reference}}\right)$ 

 $mag_{target} = mag_{reference} - 2.512 \times log(\frac{carget}{flux_{reference}})$ Example: Using SalsaJ with an aperture ra-

dius of 10 and sky annulus inner radius of 15, the Tycho-2 3695-255-1 star in the NGC957-3-V-1PlateSolved.fits image shows a star intensity count of 288497, which we can interpret as the flux of the reference star in the above formula. The V magnitude reported from Simbad for this star is 11.86. Our formula to calculate the target magnitude becomes:

 $mag_{target} = 11.86 - 2.512 \times Log\left(\frac{flux_{target}}{288497}\right)$ 

To use this formula, we substitute the measured star intensity for each target star into the  $flux_{target}$  variable in the equation. Spreadsheets easily accomplish this task.

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